

Mock JEE Main - 4 (CBT) | JEE-2024

Date: 16/01/2024

Maximum Marks: 300

Timing: 3:30 PM to 6:30 PM

Duration : 3.0 Hours

General Instructions

1. The test is of **3 hours** duration and the maximum marks is **300**.
2. The question paper consists of **3 Parts** (Part I: **Physics**, Part II: **Chemistry**, Part III: **Mathematics**). Each Part has **two** sections (Section 1 & Section 2).
3. **Section 1** contains **20 Multiple Choice Questions**. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE CHOICE** is correct.
4. **Section 2** contains **10 Numerical Value Type Questions** Out of which **ONLY 5 (any)** questions have to be attempted. You will **NOT** be allowed to attempt the sixth question. If you wish to attempt any other question apart from the five already attempted, then you will have to delete any one response from the five previously answered and then proceed to answer the new one.
The answer to each question should be **rounded off to the nearest integer**.
5. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc. inside the examination room/hall.
6. On completion of the test, the candidate must hand over the Answer Sheet to the **Invigilator** on duty in the Room/Hall. **However, the candidates are allowed to take away this Test Booklet with them.**

Marking Scheme

1. **Section – 1:** +4 for correct answer, –1 (negative marking) for incorrect answer, 0 for all other cases.
2. **Section – 2:** +4 for correct answer, –1 (negative marking) for incorrect answer, 0 for all other cases.

Name of the Candidate (In CAPITALS) :

Roll Number :

OMR Bar Code Number :

Candidate's Signature : Invigilator's Signature

PART I : PHYSICS**MARKS: 100****SECTION-1**

This section contains 20 Multiple Choice Questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE CHOICE** is correct.

1. The electric potential in a region of space is given by

$$V = Ax^2 - 3y^2 + z^2$$

Where x, y, z are coordinates of point in m . A is a positive constant.

The dimensional formula for A is:

- (A) $M L T^{-3}$ (B) $M A^{-1} T^{-3}$ (C) $M A T^{-3}$ (D) $M L^2 T^{-3}$

2. The longest wavelength of hydrogen atom in Lyman series is λ . The shortest wavelength of hydrogen atom in Balmer series will be:

- (A) $\lambda/3$ (B) $\lambda/2$ (C) 2λ (D) 3λ

3. A clock pendulum made of invar has a period of $0.5 s$ at $20^\circ C$. If the clock is used in a climate where average temperature is $30^\circ C$, how much fast or slow will the clock run in $10^6 s$?

$$(\alpha_{\text{invar}} = 1 \times 10^{-6} / ^\circ C)$$

- (A) $5 s$ (B) $3 s$ (C) $1 s$ (D) $20 s$

4. The time period of moon, revolving earth at a distance of $60R$ from earth's center, is 29 days. Time period of a satellite revolving earth at a distance of $15 R$ from earth's center will be: [R = radius of earth]

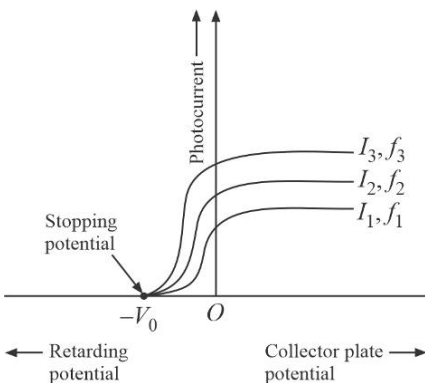
- (A) $\frac{29}{8}$ days (B) $\frac{29}{2}$ days (C) 58 days (D) 232 days

5. The distance between two plates of a parallel plate capacitor is d and its capacitance is C_0 when air is the medium between the plates. If an insulator of dielectric constant 3 and thickness $d/2$ is introduced between the plates, the capacitance of the capacitor becomes: (Take area of the insulator to be the same as that of the plate)

- (A) $2C_0$ (B) $1.5C_0$ (C) $3C_0$ (D) $2.5C_0$

SPACE FOR ROUGH WORK

6. A particle starts from rest and moves with constant acceleration. It covers x distance in first t seconds. The distance covered in the next $3t$ seconds will be:
 (A) $9x$ (B) $8x$ (C) $15x$ (D) $27x$
7. The variation of photocurrent with collector plate potential is shown below. I_1, I_2, I_3 represent different intensity and f_1, f_2, f_3 represent different frequency of incident radiation.



It can be concluded that:

- (I) $f_1 = f_2 = f_3$ (II) $I_1 = I_2 = I_3$

Choose the correct statement(s).

- (A) I (B) II (C) Both (D) Neither
8. A monoatomic ideal gas undergoes adiabatic expansion. If the final volume is 27 times the initial volume, then the ratio of initial *RMS* speed to final *RMS* speed will be:
 (A) $1 : 27$ (B) $3 : 1$ (C) $1 : 2$ (D) $27 : 1$
9. Two projectiles are projected at θ_1 and θ_2 with the horizontal with the same speed u . The difference of the horizontal distances travelled on the ground by the two will be: (Given: $\theta_1 - \theta_2 = 30^\circ$)

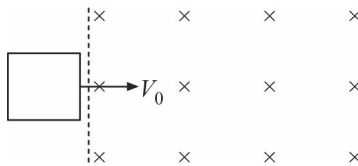
- (A) $\frac{2u^2}{g}$ (B) $\frac{u^2}{2g}$ (C) $\frac{\sqrt{3}u^2}{2g}$ (D) $\frac{u^2}{g}$

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10. The speed of a car taking a circular turn of radius 30 m on a smooth road with angle of banking $\theta = 37^\circ$ will be: (Take $g = 10\text{ m/s}^2$ and $\cos 37^\circ = 4/5$)

(A) $6\sqrt{5}\text{ m/s}$ (B) $5\sqrt{6}\text{ m/s}$ (C) 15 m/s (D) 20 m/s

11. A square coil of side a is pushed into a region containing uniform magnetic field B with constant velocity V_0 as shown below.



Determine the work done in pushing the coil into the region of magnetic field completely. Take resistance of the coil to be R .

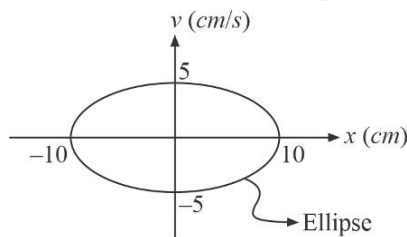
(A) $\frac{B^2 a^2 V_0^2}{R}$ (B) $\frac{B^2 a^3 V_0}{R}$ (C) $\frac{2B^2 a^2 V_0^2}{R}$ (D) $\frac{2B^2 a^3 V_0}{R}$

12. In a metallic conductor, under the effect of a given applied electric field (\vec{E}), which of the following statement is correct?

(A) Current density (\vec{J}) is opposite to \vec{E}
 (B) Magnitude of current density is doubled if number of free electrons per unit volume (n) is halved
 (C) Magnitude of current density remains the same if cross-sectional area (A) of the conductor is halved
 (D) Magnitude of current density would remain the same if protons were free charge carriers instead of free electron

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13. Graph between velocity of an object oscillating simple harmonically along x -axis and its displacement from the mean position is shown below. Find out its time period.



- (A) 4π seconds (B) 2π seconds (C) π seconds (D) $\frac{\pi}{2}$ seconds

14. The electric field part of an electromagnetic wave propagating in a medium is represented by

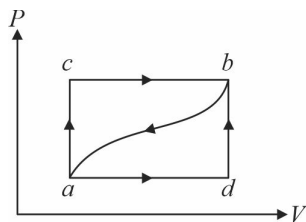
$$E_z = 5 \sin \left[-(\pi \times 10^{-2} \text{ rad/m})x - (2\pi \times 10^6 \text{ rad/s})t + \frac{\pi}{6} \right] \frac{N}{C}$$

Choose the correct statement.

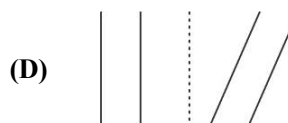
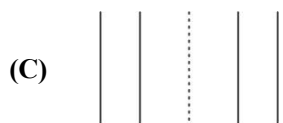
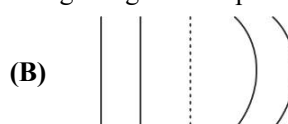
- (A) Refractive index of the medium is 1.5
 (B) Wavelength of the wave in medium is 2 nm
 (C) The maximum value of magnetic field in the medium is $2.5 \mu T$
 (D) The propagation speed of the wave in the medium is $C = 3 \times 10^8 \text{ m/s}$
15. **Assertion A:** If the radius of the earth were to expand due to its geothermal pressure, then the duration of the day would be less than 24 hours.
Reason R: If net external torque on a body about an axis is zero, then its angular momentum about that axis remains conserved.
- (A) A is not correct but R is correct
 (B) Both A and R are correct and R is the correct explanation of A
 (C) Both A and R are correct but R is NOT the correct explanation of A
 (D) A is correct but R is not correct
16. An experiment is performed with a screw gauge, which has main scale division of 0.5 mm and its circular scale has 50 divisions. While measuring the diameter of a spherical ball, cap end is after 20^{th} marking of main scale and 35^{th} division of circular scale coincides with the reference line. The diameter of the ball will be:
- (A) 20.35 mm (B) 20.70 mm (C) 10.35 mm (D) 10.70 mm

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17. When a system is taken from state 'a' to state 'b' along the path 'acb', it is found that a quantity of heat $Q = 200 \text{ J}$ is absorbed by the system and a work $W = 80 \text{ J}$ is done by it. Along the path 'adb', $Q = 144 \text{ J}$ is absorbed by the system. The work done along the path 'abd' is:



- (A) 6 J (B) 12 J (C) 18 J (D) 24 J
18. In each option, wavefront, shown as dark line, travelling left to right is incident on optical instrument shown as dotted line. Which option represents bending of light while passing through a prism?



SPACE FOR ROUGH WORK

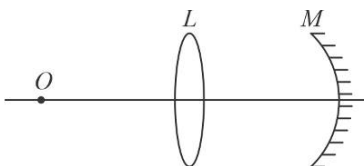
19. Work W is required to form a bubble of volume V from a given solution. What amount of work is required to be done to form bubble of volume $2V$?
- (A) W (B) $2W$ (C) $2^{1/3}W$ (D) $4^{1/3}W$
20. (I) In a semiconductor diode, the diffusion of majority charge carriers due to concentration difference gives rise to diffusion current from p-side to n -side.
- (II) In a semiconductor diode, a layer of positive charge on p-side and a layer of negative charge on n-side are developed in the depletion region.
- (III) In a photodiode, the junction under reverse bias, is illuminated with light with energy ($h\nu$) greater than the energy gap (E_g) giving rise to reverse current.
- (IV) When gallium arsenide diode is under forward bias, free electrons and holes move towards the junction and recombine, releasing visible energy.
- Choose the correct statement(s).
- (A) I, II, III and IV (B) I, II and III (C) I, III and IV (D) I and II

SPACE FOR ROUGH WORK

SECTION-2

Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted. The answer to each question should be rounded off to the nearest integer.

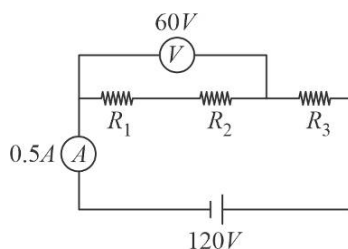
1. A point, 'O' is placed in front of a convex lens L and a concave mirror M as shown below.



- It is observed that the final image formed by the system is at the location of the object. By what distance (in cm) should the mirror be displaced such that the position of the final image remains undisturbed? (Object 'O' and Lens 'L' remain undisturbed. Take focal length of the mirror to be 10 cm).
2. Find percentage change in earth's gravitational field as one moves from its surface to an attitude of 32 km . (Take radius of the earth $R = 6.4 \times 10^6\text{ m}$). Write answer to the nearest integer.
3. Two point masses each carrying charge Q are fixed at two of the vertices of an equilateral triangle of sides l . At the third vertex, a third particle of mass m and also carrying charge Q is placed. When it is released, it's initial acceleration (in m/s^2) will be _____.

(Take $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \frac{Nm^2}{C^2}$, $m = 3\sqrt{3}\text{ gm}$, $Q = 0.1\text{ }\mu\text{C}$ and $l = 10\text{ cm}$).

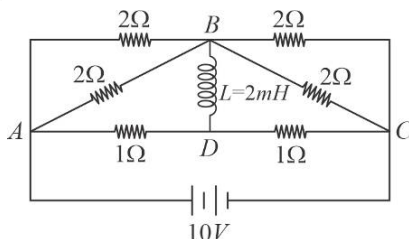
4. In the circuit shown, voltmeter, ammeter and battery are ideal. The readings of voltmeter and ammeter are as shown in figure. The resistance R_3 will be _____ (in ohm).



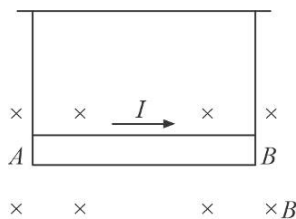
5. Two identical metal balls with charges Q and $3Q$ are brought in contact and then separated to a large distance. The percentage loss of energy in the process will be _____.

SPACE FOR ROUGH WORK

6. Two deuterons undergo nuclear fusion to form a Helium nucleus. Energy released in the process in MeV will be _____. (Binding energy per nucleon for deuteron is $1 MeV$ and for helium is $7 MeV$)
7. One end of a rod is fixed at the origin and is free to rotate about z -axis. Its moment of Inertia about this axis is $2 \times 10^3 gm \cdot cm^2$. A force $\vec{F} = 10^{-1} N \hat{i}$ acts on a point $(2 cm, 2 cm, 0)$, on the rod. Initial angular acceleration of the rod will be _____ (in rad/s^2).
8. In the circuit shown below, the current through the battery will be _____ (in ampere). (Assume the circuit to be closed for a long time).



9. A particle is executing simple harmonic motion. The ratio of kinetic energy to potential energy of the particle, when its displacement from mean position, is one-third of its amplitude is a/b expressed in the lowest form. Write $(a + b)$ as your answer. (Assume potential energy at mean position is zero).
10. A straight wire AB of mass $25 g$ and length $50 cm$ is suspended by a pair of strings in uniform magnetic field of magnitude $0.5 T$ as shown below. The magnitude of the current required in the wire to remove tension in the strings is _____ Ampere. (Take $g = 10 m/s^2$)



SPACE FOR ROUGH WORK

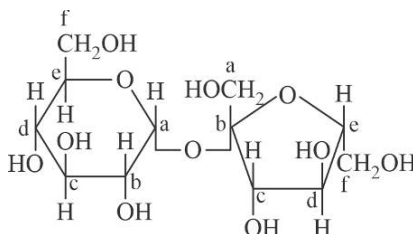
PART II : CHEMISTRY**MARKS: 100****SECTION 1**

This section contains 20 Multiple Choice Questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE CHOICE** is correct.

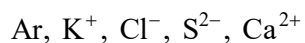
- Which of the following is not correct?

(A) $[\text{Fe}(\text{CN})_6]^{3-}$ is weakly paramagnetic (B) $[\text{Fe}(\text{CN})_6]^{4-}$ is strongly paramagnetic

(C) $[\text{Ni}(\text{CN})_4]^{2-}$ is diamagnetic (D) $[\text{NiCl}_4]^{2-}$ is paramagnetic
- Structure of a disaccharide formed by glucose and fructose is given below. Identify anomeric carbon atoms in monosaccharide units.



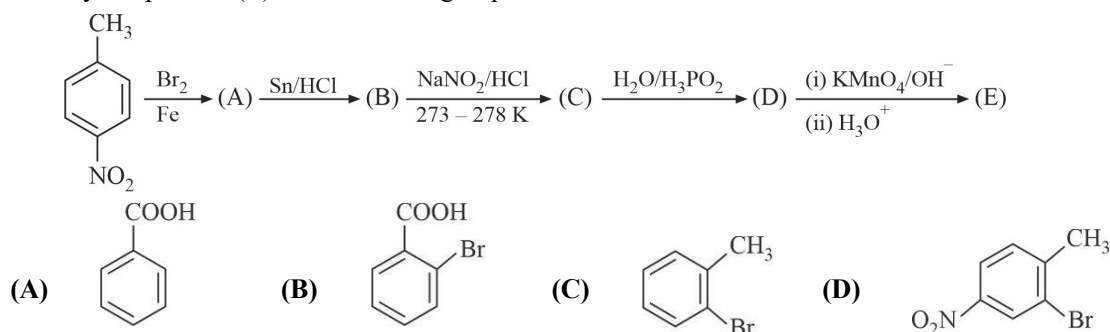
- (A) 'a' carbon of glucose and 'a' carbon of fructose
 (B) 'a' carbon of glucose and 'e' carbon of fructose
 (C) 'a' carbon of glucose and 'b' carbon of fructose
 (D) 'f' carbon of glucose and 'f' carbon of fructose
- The correct decreasing order of atomic size among the following species is:



- (A) $\text{Ca}^{2+} > \text{K}^+ > \text{Ar} > \text{Cl}^- > \text{S}^{2-}$ (B) $\text{K}^+ > \text{Ca}^{2+} > \text{Cl}^- > \text{Ar} > \text{S}^{2-}$
- (C) $\text{S}^{2-} > \text{Cl}^- > \text{Ar} > \text{K}^+ > \text{Ca}^{2+}$ (D) $\text{S}^{2-} > \text{Ar} > \text{Cl}^- > \text{Ca}^{2+} > \text{K}^+$

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4. Identify the product (E) in the following sequence of reactions:



5. In case, nitrogen and sulphur both are present in an organic compound, sodium thiocyanate is formed. If sodium fusion is carried out with excess of sodium, sodium thiocyanate decomposes. Which of the following compounds is/are present in the extract after decomposition?

- (A) Only NaCN (B) Only Na₂S
 (C) Both NaCN & Na₂S (D) (SCN)₂

6. An alcohol gives Lucas test within 5 minutes. The structure of alcohol having mass 0.22 g which liberates 56 ml of CH₄ at STP on treatment with CH₃ MgI will be:

- (A) $\text{CH}_3-\text{CH}(\text{OH})-\text{CH}_3$ (B) $\text{CH}_3\text{CH}_2-\text{CH}(\text{OH})-\text{CH}_3$
 (C) $(\text{CH}_3)_2\text{CH}-\text{CH}(\text{OH})-\text{CH}_3$ (D) $(\text{CH}_3)_3\text{C}-\text{CH}(\text{OH})-\text{CH}_3$

SPACE FOR ROUGH WORK

7. Match the column-I to column-II.

Column-I (Complex)		Column-II (Structure and magnetic moment)	
(i)	$[\text{Ag}(\text{CN})_2]^-$	(p)	Square planar and 1.73 BM
(ii)	$[\text{Cu}(\text{CN})_4]^{3-}$	(q)	Linear, zero
(iii)	$[\text{Cu}(\text{NH}_3)_4]^{2+}$	(r)	Octahedral, zero
(iv)	$[\text{Fe}(\text{CN})_6]^{4-}$	(s)	Tetrahedral, zero

- (A) (i) – (q), (ii) – (s), (iii) – (p), (iv) – (r) (B) (i) – (q), (ii) – (s), (iii) – (r), (iv) – (p)
 (C) (i) – (s), (ii) – (q), (iii) – (p), (iv) – (r) (D) (i) – (q), (ii) – (r), (iii) – (s), (iv) – (p)

8. **Statement I:** Both 12 g of carbon and 27 g of aluminium contain 6.022×10^{23} atoms.

Statement II: Gram atomic mass of an element contains Avogadro number of atoms.

- (A) Both statement I and statement II are correct and statement II is the correct explanation of the statement I
 (B) Both statement I and statement II are correct, but statement II is not the correct explanation of statement I
 (C) Statement I is correct, but statement II is wrong
 (D) Statement I is wrong, but statement II is correct
9. Highest oxidation state of manganese in fluoride is +4 (MnF_4) but highest oxidation state in oxides is +7 (Mn_2O_7) because.
- (A) Fluorine is more electronegative than oxygen
 (B) Fluorine does not possess d-orbitals
 (C) Fluorine stabilizes lower oxidation state
 (D) In covalent compounds fluorine can form single bond only while oxygen forms double bond

SPACE FOR ROUGH WORK

10. Match the following compound with their equivalent weights and choose the correct code given below.

Column-I (Compound)		Column-II (Equivalent weight)	
(i)	$\text{Na}_3\text{PO}_4 : \text{Na}_3\text{PO}_4 + 2\text{HCl} \rightarrow \text{NaH}_2\text{PO}_4 + 2\text{NaCl}$	(p)	51.67
(ii)	$\text{Ca}_3(\text{PO}_4)_2 : \text{CaCl}_2 + \text{Na}_3\text{PO}_4 \rightarrow \text{Ca}_3(\text{PO}_4)_2 + \text{NaCl}$	(q)	63
(iii)	$\text{Na}_2\text{CO}_3 : \text{Na}_2\text{CO}_3 + \text{HClO}_4 \rightarrow \text{NaHCO}_3 + \text{NaClO}_4$	(r)	82
(iv)	$\text{H}_2\text{C}_2\text{O}_4 \cdot 2\text{H}_2\text{O} : \text{H}_2\text{C}_2\text{O}_4 \cdot 2\text{H}_2\text{O} + \text{CaO} \rightarrow \text{CaC}_2\text{O}_4 + \text{H}_2\text{O}$	(s)	106

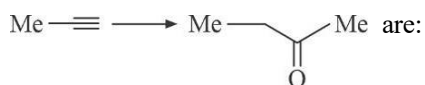
(Atomic mass : Na = 23, P = 31, O = 16, Ca = 40, C = 12, H = 1).

- (A) (i) – (q), (ii) – (r), (iii) – (s), (iv) – (p) (B) (i) – (p), (ii) – (r), (iii) – (s), (iv) – (q)
 (C) (i) – (r), (ii) – (p), (iii) – (q), (iv) – (s) (D) (i) – (r), (ii) – (p), (iii) – (s), (iv) – (q)

11. $\text{CH}_3\text{COOH} \xrightarrow{\text{LiAlH}_4} (\text{X}) \xrightarrow[573 \text{ K}]{\text{Cu}} (\text{Y}) \xrightarrow[\text{NaOH}]{\text{Dilute}} (\text{Z})$.


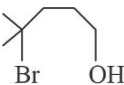
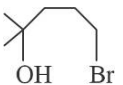

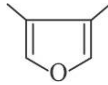
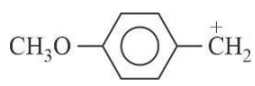
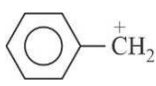
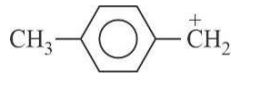
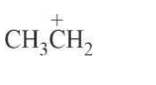
In the above reaction, (Z) is:

- (A) Aldol (B) Ketol (C) Acetol (D) Butanol
12. The reagents use to carry out the following conversion



- (A) $\text{HgSO}_4 / \text{dil. H}_2\text{SO}_4$ (B) $\text{BH}_3, \text{H}_2\text{O}_2 / \text{NaOH}$
 (C) $\text{NaNH}_2 / \text{CH}_3\text{I}, \text{HgSO}_4 / \text{dil. H}_2\text{SO}_4$ (D) $\text{OsO}_4, \text{HIO}_4$
13. Which of the following will produce a buffer solution when mixed in equal volumes?
- (A) $0.1 \text{ mol dm}^{-3} \text{ NH}_4\text{OH}$ and $0.1 \text{ mol dm}^{-3} \text{ HCl}$
 (B) $0.05 \text{ mol dm}^{-3} \text{ NH}_4\text{OH}$ and $0.1 \text{ mol dm}^{-3} \text{ HCl}$
 (C) $0.1 \text{ mol dm}^{-3} \text{ NH}_4\text{OH}$ and $0.05 \text{ mol dm}^{-3} \text{ HCl}$
 (D) $0.1 \text{ mol dm}^{-3} \text{ CH}_3\text{COONa}$ and $0.1 \text{ mol dm}^{-3} \text{ NaOH}$

SPACE FOR ROUGH WORK

14. The source of nitrogen in Gabriel synthesis of amines is:
- (A) Sodium azide, NaN_3 (B) Sodium nitrite, NaNO_2
 (C) Potassium cyanide, KCN (D) Potassium phthalimide, $\text{C}_6\text{H}_4(\text{CO})_2\text{N}^- \text{K}^+$
15. **Assertion A:** Bond order in a molecule can assume any value, positive or integral or fractional including zero.
Reason R: It depends upon the number of electrons in the bonding and anti-bonding orbitals.
- (A) If both A and R are correct and R is the correct explanation of the A
 (B) If both A and R are correct but R is not the correct explanation of the A
 (C) If A is correct but R is incorrect
 (D) If both A and R are incorrect
16.  on reaction with cold conc. HBr gives:
- (A)  (B)  (C)  (D) 
17. The dipole moment of HBr is $2.6 \times 10^{-30} \text{ C m}$ and the interatomic spacing is 1.41 \AA . The percentage of ionic character in HBr is:
- (A) 10.5 (B) 11.5 (C) 12.5 (D) 13.5
18. Among the following which has reddish brown colour.
- (A) $\text{Al}(\text{OH})_3$ (B) $\text{Fe}(\text{OH})_3$ (C) CuS (D) HgS
19. Relative stabilities of the following carbocations will be in the order.
- (I)  (II) 
 (III)  (IV) 
- (A) $\text{I} < \text{II} < \text{III} < \text{IV}$ (B) $\text{IV} < \text{III} < \text{II} < \text{I}$ (C) $\text{IV} < \text{II} < \text{III} < \text{I}$ (D) $\text{II} < \text{IV} < \text{III} < \text{I}$
20. The first ionization potential of Na, Mg and Si are respectively 496, 737 and 786 kJ mol^{-1} . The first ionization potential of Al will be closer to:
- (A) 760 kJ mol^{-1} (B) 575 kJ mol^{-1} (C) 801 kJ mol^{-1} (D) 419 kJ mol^{-1}

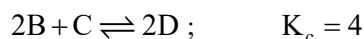
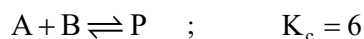
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SECTION-2

Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted. The answer to each question should be rounded off to the nearest integer.

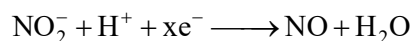
1. If the number of revolutions made by electron in 1 second in H atom in its n^{th} orbit is twice of the number of revolutions made by electron in 1 second in the 2^{nd} orbit of He^+ ion atom, then n is:

2. Consider the following reversible reactions:



Hence, equilibrium constant (K_c) for the reaction $\text{A} + \text{D} \rightleftharpoons \text{P} + \frac{\text{C}}{2}$ is:

3. Consider the following redox reaction:



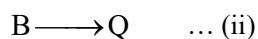
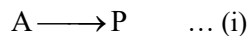
The value of x is:

4. An alkane of molar mass 72 gram/mol gives a single mono-chloro derivative and two di-chloro derivatives on chlorination. The number of 1° hydrogen in the alkane are x. The value of $x/3$ is _____.

5. A 0.4 molal aqueous solution of Na_xA has freezing point of -3.72°C . If K_f of water is $1.86 \text{ K kg mol}^{-1}$, the value of x is: (Assume complete ionization of salt Na_xA).

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6. Consider the following two first order reactions:



Reaction (i) is 75% complete in 4 hrs while reaction, (ii) takes 16 hrs for same 75% completion of reaction under identical conditions. By how many hours, half-life of (ii) is greater than the half-life of (i)?

7. On treatment of 100 mL of 0.1 M solution of the complex $\text{CoCl}_3 \cdot 6\text{NH}_3$ with excess of AgNO_3 , 4.305 g of AgCl was obtained. What is the positive charge present on the complex ion?
8. Equivalent conductance of 0.2 M aqueous solution of a weak monobasic acid (HA) is $10 \text{ S cm}^2 \text{ equiv}^{-1}$ and that at infinite dilution is $200 \text{ S cm}^2 \text{ equiv}^{-1}$. Hence, pH of this solution is:
9. An ideal gas is allowed to expand against a constant pressure of 2 bar from 10 L to 50 L. The work done by the gas is $-x\text{kJ}$. Find the value of x. [Given that 1 L bar = 100 J]
10. How many electrons are present in anti-bonding MO's in CN^+ ion ?

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PART III : MATHEMATICS**MARKS: 100****SECTION 1**

This section contains 20 Multiple Choice Questions. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE CHOICE is correct.

- The complex number z satisfies $z + |z| = 2 + 8i$. The value of $|z|$ is:
 (A) 10 (B) 13 (C) 17 (D) 23
- The line which contains all points (x, y, z) , which are of the form $(x, y, z) = (2, -2, 5) + \lambda(1, -3, 2)$ intersects the plane $2x - 3y + 4z = 163$ at P and the YZ plane at Q . If the distance PQ is $a\sqrt{b}$ where $a, b \in N$ and $a > 3$ then $(a + b)$ equals:
 (A) 23 (B) 95 (C) 27 (D) 25
- The remainder when $4^{87} + 6^{87}$ is divided by 25, is:
 (A) 19 (B) 20 (C) 21 (D) 22
- Let ABC be a triangle whose centroid is G , orthocenter is H and circumcentre is the origin O . If D is any point in the plane of the triangle such that no three of O, A, B, C and D are collinear and they satisfy the relation $\overrightarrow{AD} + \overrightarrow{BD} + \overrightarrow{CH} + 3\overrightarrow{HG} = \lambda\overrightarrow{HD}$, then the value of the scalar λ is:
 (A) 8 (B) 6 (C) 4 (D) 2
- $x - 2y + 4 = 0$ is a common tangent to $y^2 = 4x$ and $\frac{x^2}{4} + \frac{y^2}{b^2} = 1$. Then the value of b and the other common tangent are given by:
 (A) $b = \sqrt{3}; x + 2y + 4 = 0$ (B) $b = 3; x + 2y + 4 = 0$
 (C) $b = \sqrt{3}; x + 2y - 4 = 0$ (D) $b = \sqrt{3}; x - 2y - 4 = 0$

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6. If $\int \frac{(2x+3)dx}{x(x+1)(x+2)(x+3)+1} = C - \frac{1}{f(x)}$ where $f(x)$ is of the form of $ax^2 + bx + c$ then $(a + b + c)$ equals:
 (A) 4 (B) 5 (C) 6 (D) 7
7. In a game of chance a player throws a pair of dice and scores points equal to the difference between the numbers on the two dice. Winner is the person who scores exactly 5 points more than his opponent. If two players are playing this game only one time, then the probability that neither of them win, is:
 (A) $\frac{1}{54}$ (B) $\frac{1}{108}$ (C) $\frac{53}{54}$ (D) $\frac{107}{108}$
8. $\int \frac{1}{\sqrt[3]{x^2} \sqrt[3]{(2+3x)^4}} dx$ is:
 (A) $\frac{3}{2} \sqrt[3]{\frac{x}{2+3x}} + c$ (B) $\frac{1}{2} \sqrt[3]{\frac{x}{2+3x}} + c$
 (C) $\frac{1}{2} \left(\frac{x}{2+3x} \right)^2 + c$ (D) $\frac{2}{3} \sqrt[3]{\frac{x}{2+3x}} + c$
9. Coordinates of a point Q lying on the line $\frac{x-2}{4} = \frac{y-1}{3} = \frac{z+1}{2}$ such that the line joining the points Q and $P(3, 1, 5)$ is parallel to the plane $4x - 2y + z = 0$, is:
 (A) $\left(\frac{16}{3}, \frac{7}{2}, \frac{2}{3} \right)$ (B) $(2, 1, -1)$ (C) $\left(4, \frac{5}{2}, 0 \right)$ (D) $\left(3, \frac{7}{4}, \frac{-1}{2} \right)$
10. Let p be a prime number of the form $6I + 1$, where I is an integer, then $\prod_{k=1}^{p-1} \sin\left(\frac{k\pi}{p}\right)$ is equal to:
 (A) $\frac{p}{2^p}$ (B) $\frac{p-1}{2^{p-1}}$ (C) $\frac{1}{2^{p-1}}$ (D) $\frac{p}{2^{p-1}}$

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11. If the matrices $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{bmatrix}$, $B = \text{adj } A$ and $C = 3A$, then $\frac{|\text{adj } B|}{|C|}$ is equal to:
(A) 16 (B) 2 (C) 72 (D) 8
12. If the coefficient of variation of some observations is 60 and their standard deviation is 20, then their mean is:
(A) 35 (B) 34 (C) 30.33 (D) 33.33
13. The locus of the middle point of the chord of the circle $x^2 + y^2 = 1$ such that the segment of the chord on the parabola $y = x^2 - x$ subtends a right angle at the origin, is a circle whose centre and radius respectively are:
(A) $(1, 1)$ and $\sqrt{2}$ (B) $(1, 1)$ and 2
(C) $\left(\frac{1}{2}, \frac{1}{2}\right)$ and $\frac{1}{2}$ (D) $\left(\frac{1}{2}, \frac{1}{2}\right)$ and $\frac{1}{\sqrt{2}}$
14. Find the number of ways in which four different toys and five indistinguishable marbles can be distributed between Amar, Akbar and Anthony, if each child receives at least one toy and one marble.
(A) 216 (B) 215 (C) 305 (D) 205
15. Six persons A, B, C, D, E and F are to be seated at a circular table. The number of ways this can be done if A must have either B or C on his immediate right and B must have either C or D on his immediate right is:
(A) 36 (B) 12 (C) 24 (D) 18

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16. Let $f(x) = 1 + 4x - x^2, \forall x \in R$, $g(x) = \begin{cases} \max\{f(t) : x \leq t \leq (x+1) : 0 \leq x < 3\} \\ \min\{(x+3) : 3 \leq x \leq 5\} \end{cases}$, then $g(x)$ is:
- (A) Continuous $\forall x \in [0, 5]$ (B) Continuous $\forall x \in [0, 5] - \{3\}$
 (C) Continuous $\forall x \in [0, 4]$ (D) Discontinuous at all integral values of x
17. Given $(1 - 2x + 5x^2 - 10x^3)(1 + x)^n = 1 + a_1x + a_2x^2 + \dots$ and that $a_1^2 = 2a_2$, then the value of n is:
- (A) 6 (B) 2 (C) 5 (D) 3
18. Given the relation $R = \{(2, 3), (3, 4)\}$ on the set $\{2, 3, 4\}$. The number of minimum number of ordered pairs to be added to R so that R is reflexive and symmetric:
- (A) 4 (B) 5 (C) 7 (D) 6
19. Let $\vec{r} = (\vec{a} \times \vec{b}) \sin x + (\vec{b} \times \vec{c}) \cos y + 2(\vec{c} \times \vec{a})$, where $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors. If \vec{r} is perpendicular to $\vec{a} + \vec{b} + \vec{c}$, then minimum value of $x^2 + y^2$ is:
- (A) π^2 (B) $\frac{\pi^2}{4}$ (C) $\frac{5\pi^2}{4}$ (D) $\frac{13\pi^2}{4}$
20. Let a, b, c and d be positive real numbers with $a < b < c < d$. Given that a, b, c, d are the first four terms an A.P. and a, b, d are the first three terms of a G.P., the ratio $\frac{ad}{bc}$ is equal to:
- (A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{3}{4}$ (D) $\frac{4}{5}$

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SECTION-2

Section 2 contains 10 Numerical Value Type Questions Out of which ONLY 5 (any) questions have to be attempted. The answer to each question should be **rounded off to the nearest integer**.

1. The equation of a line through the mid-point of the sides AB and AD of rhombus $ABCD$, whose one diagonal is $3x - 4y + 5 = 0$ and one vertex is $A(3, 1)$, is $ax + by + c = 0$. Find the absolute value of $(a + b + c)$ where a, b, c are integers expressed in lowest form.
2. Find $b, b \geq 2$ satisfying the equations $3 \log_b(\sqrt{x} \log_b x) = 56$ and $\log_{\log_b x}(x) = 54$, where $x > 1$.
3. All the five digit numbers in which each successive digit exceeds its predecessor are arranged in the increasing order of their magnitude. Then the sum of digit of the 97th number is:
4. The area bounded by $y = xe^{|x|}$ and line $|x| = 1, y = 0$ is A sq. units then A equals:
5. $y = x + 2$ is any tangent to the parabola $y^2 = 8x$. The ordinate of the P on this tangent such that the other tangent from it which is perpendicular to it is:
6. If L be the length of common tangent to the ellipse $\frac{x^2}{25} + \frac{y^2}{4} = 1$ and the circle $x^2 + y^2 = 16$ intercepted by the coordinate axis then $\frac{\sqrt{3}L}{2}$ is:
7. The number of normal(s) of a rectangular hyperbola which can touch its conjugate is equal to _____.
8. Two lines $L_1 : x = 5, \frac{y}{4 - \alpha} = -z$ and $L_2 : x = \alpha, \frac{y}{-2} = \frac{z}{2 - \alpha}$ coplanar. Then, sum of all possible values of α is:
9. A conic C satisfies the differential equation $(1 + y^2)dx - xydy = 0$ and passes through the point $(1, 0)$. An ellipse E which is confocal with C has its eccentricity as $\sqrt{2/3}$. The angle of intersection of the curves C and E is k (in degrees), then k is:
10. If the range of function $f(x) = \left(\frac{\pi}{\sqrt{2}} + \sin^{-1} \alpha \right) x^2 + 2(\sin^{-1} \beta)x + \frac{\pi}{\sqrt{2}} - \sin^{-1} \alpha$ is $[0, \infty)$, then the maximum value of $(\alpha^2 + \beta^2 + \alpha^2 \beta^2)$ is equal to:

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